Cardinality Analysis and its Applications: from Glasgow Haskell Compiler to Sharded Blockchains

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Part I
Cardinality Analysis for Haskell Programs

Joint work with Dimitrios Vytiniotis and Simon Peyton Jones, presented at POPL'14

Optimising compiler in a nutshell

$$p_1$$

Optimising compiler in a nutshell

$$\begin{tikzcd}
p_1 \arrow[r] \arrow[d] & c_1 \\
p_2 \arrow[r] & c_2
\end{tikzcd}$$

$$c_2 \simeq c_1$$

$$c_2 \preceq c_1$$

$$
\begin{tikzcd}
p_1 \arrow[r] \arrow[d, "\mathcal{A}(p_1)"'] & c_1 \\
p_2 \arrow[r] & c_2
\end{tikzcd}
$$

Annotating static analysis

$$\mathcal{A}$$

A story of
three program optimisations

Optimisation 1

```
f1, f2 :: [Int] -> Int
```

Which function is better to run?

Better

```
f1 xs = let ys = map costly xs
        in squash (\n -> sum (map (+ n) ys))
```

if invoked *more than once* by squash

```
f2 xs = squash (\n -> sum (map (+ n) (map costly xs)))
```

Better

if invoked *at most* once by squash

Optimisation 1

```
squash1, squash2 :: (Int -> Int) -> Int

squash1 k = sum (map k [1..10])

squash2 k = 2 * (k 0)

f1 xs = let ys = map costly xs
        in squash (\n. sum (map (+ n) ys))

f2 xs = squash (\n. sum (map (+ n) (map costly xs)))
```

Need to know:
how many times
a function is called.

(call cardinality)

Optimisation 2

“worker-wrapper” split

```
f x = case x of (p,q) -> <cbody>
```

Optimisation 2

“worker-wrapper” split

“wrapper”, usually inlined on-site

```
f x = case x of (p,q) -> fw p q

fw p q = <cbody>
```

“worker”

Optimisation 2

“worker-wrapper” split

What if `q` is *never* used in `<cbody>`?

```
f x = case x of (p,q) -> fw p

fw p   = <cbody>
```

Don’t have to pass `q` to `fw`!

Which parts of
a data structure are
certainly *not* used?

(absence)

Optimisation 3

smart memoisation

```
f :: Int -> Int -> Int
f x c = if x > 0  then c + 1 else
        if x == 0 then 0     else c - 1

g y = f y (costly y)
```

Will be used exactly once:
no need to memoize!

Which parts
of a data structure
are used *no more than once*?

(thunk cardinality)

Cardinality Analysis

- Call cardinality
- Absence
- Thunk cardinality

Usage demands

(*how* a value is used)

			call demand
Usage demands	d	$::=$	$\boxed{C^n(d)} \mid U(d_1^\dagger, d_2^\dagger) \mid U$
Cardinality demands	$d^\dagger$	$::=$	$A \mid n * d$
Usage cardinalities	n	$::=$	$1 \mid \omega$

tuple demand

Usage demands	d	$::=$	$C^n(d) \mid \boxed{U(d_1^\dagger, d_2^\dagger)} \mid U$
Cardinality demands	$d^\dagger$	$::=$	$A \mid n * d$
Usage cardinalities	n	$::=$	$1 \mid \omega$

general demand

Usage demands	d	$::=$	$C^n(d) \mid U(d_1^\dagger, d_2^\dagger) \mid \boxed{U}$
Cardinality demands	$d^\dagger$	$::=$	$A \mid n * d$
Usage cardinalities	n	$::=$	$1 \mid \omega$

Usage demands	d	$::=$	$C^n(d) \mid U(d_1^\dagger, d_2^\dagger) \mid U$
Cardinality demands	$d^\dagger$	$::=$	absent value $\boxed{A} \mid n * d$
Usage cardinalities	n	$::=$	$1 \mid \omega$

Usage demands	d	$::=$	$C^n(d) \mid U(d_1^\dagger, d_2^\dagger) \mid U$
			used at most n times
Cardinality demands	$d^\dagger$	$::=$	$A \mid \boxed{n * d}$
Usage cardinalities	n	$::=$	$1 \mid \omega$

Usage Types

(*how* a function uses its arguments)

$$\texttt{wurble1 ::}\ \boxed{\omega * U} \to C^{\omega}(C^{1}(U)) \to \bullet$$

$$\texttt{wurble1 a g = g 2}\ \boxed{\texttt{a}}\ \texttt{+ g 3}\ \boxed{\texttt{a}}$$

$$\texttt{wurble1 ::}\quad \omega * U \to \boxed{C^{\omega}(C^{1}(U))} \to \bullet$$

$$\texttt{wurble1 a g =}\ \boxed{\texttt{g}}\ \texttt{2 a +}\ \boxed{\texttt{g}}\ \texttt{3 a}$$

wurble2 :: $\boxed{\omega * U} \to C^1(C^\omega(U)) \to \bullet$

```
wurble2 a g = sum (map (g a) [1..1000])
```

```
wurble2 ::
```
$$\omega * U \to \boxed{C^1(C^\omega(U))} \to \bullet$$

```
wurble2 a g = sum (map (g a) [1..1000])
```

$\texttt{f ::} \quad 1{*}U(1{*}U, A) \to \bullet$

```
f x = case x of (p, q) -> p + 1
```

Usage type
depends on a usage context!

(*result* demand determines *argument* demands)

Two Types of Modular Program Analyses

- Forward analysis
  - “Run” the program with *abstract* input and infer the *abstract* result;
  - Examples: sign analysis, interval analysis, type checking/inference.
- Backwards analysis
  - From the expected *abstract* result of the program infer the abstract values of its inputs.

Backwards Analysis

Infers demand type basing on a context

$$P \mapsto e \downarrow d \Rightarrow \langle \tau \,;\, \varphi \rangle$$

$$P \mapsto e \downarrow d \Rightarrow \langle \tau \,;\, \varphi \rangle$$

- *P - signature environment,* maps *some* of free variables of *e* to their demand signatures (*i.e.,* keeps some contextual information)
- *d - usage demand,* describes the degree to which *e* is evaluated
- τ - *demand type,* usages that *e* places on its arguments
- φ - *fv-usage,* usages that *e* places on its free variables

$$C^1(U)$$

$$e = \lambda x\ .\ \texttt{case}\ x\ \texttt{of}\ (p, q) \to (p, f\ \texttt{True})$$

$$e = \lambda x \;.\; \texttt{case}\ x\ \texttt{of}\ (p, q) \rightarrow (p, \boxed{f\ \texttt{True}})$$

$$\epsilon \vdash\!\!\blacktriangleright e \downarrow C^1(U) \Rightarrow \langle \underbrace{1 * U(\omega * U, A) \rightarrow \bullet}_{\tau} ; \underbrace{\{\boxed{f \mapsto 1 * C^1(U)}\}}_{\varphi} \rangle$$

Each function is a
backwards demand transformer
it transforms a *context* demand to
argument demands and *fv*-demands.

How many context demands are there?

∞

We cannot compute best argument demands
for *all* contexts:
need to *approximate*.

Demand Lattice

$\top = \omega * U$

$1 * U$ $\quad$ $\omega * U(\omega * U, A)$ $\quad$ $\omega * C^{\omega}(U)$

$1 * U(A, \omega * C^{1}(U))$ $\quad$ $1 * U(\omega * U, A)$

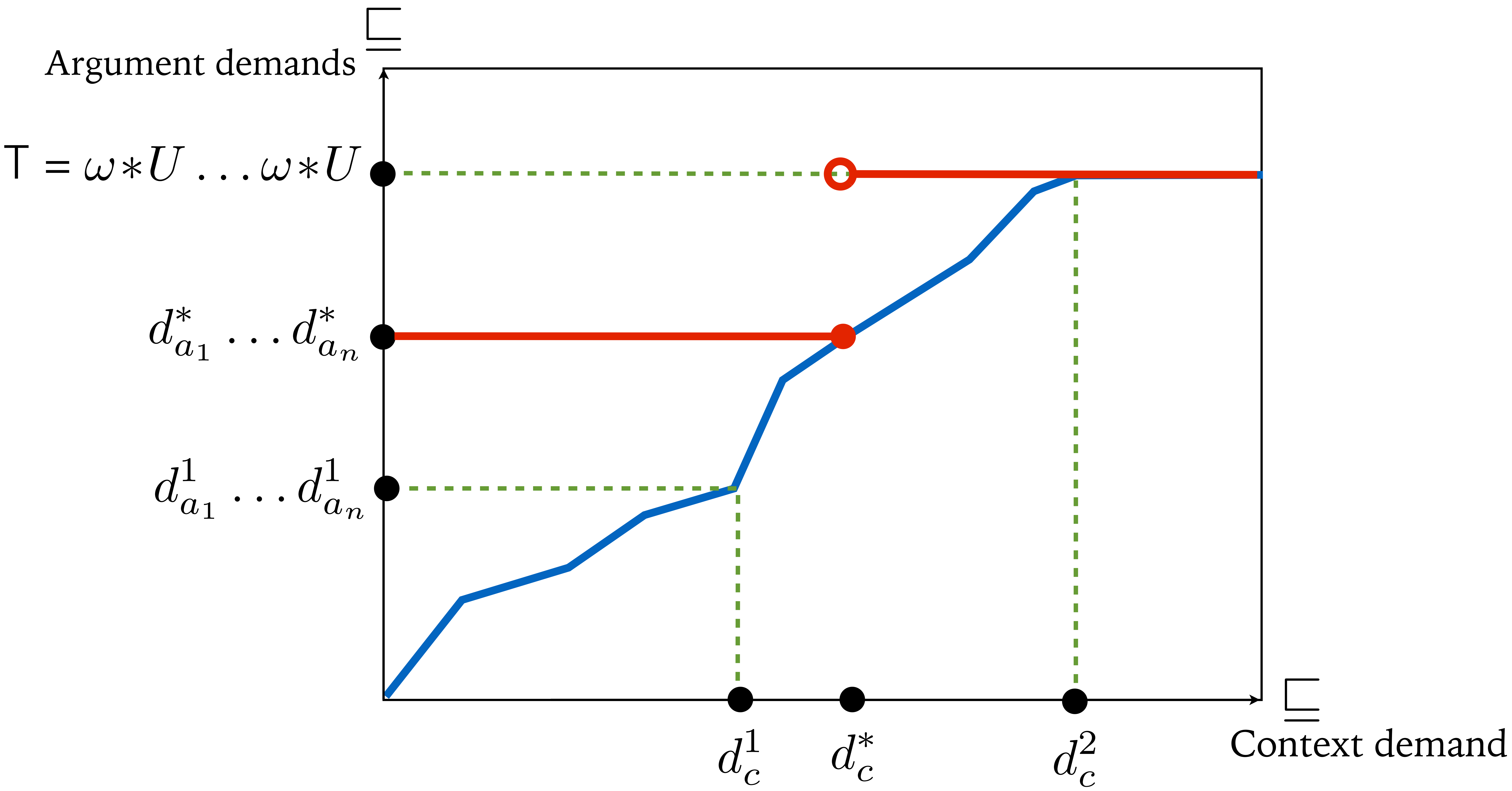
$\omega * C^{\omega}(U(A, \omega * U))$

$1 * U(A, A)$ $\quad$ $1 * C^{\omega}(U(A, \omega * U))$

$\perp = A$

Each function is
a monotone backwards demand transformer.

Exploiting demand monotonicity



Analysis-based annotations

$$P \vdash\!\!\blacktriangleright e \downarrow d \Rightarrow \langle \tau \; ; \varphi \rangle$$

Elaboration

$$P \mapsto e \downarrow d \Rightarrow \langle \tau \, ; \varphi \rangle \rightsquigarrow \mathsf{e}$$

- **let**-bindings in **e** are annotated with **m** ∈ **{0, 1, ω}** to indicate how often the let-binding is evaluated;
- Each Lambda $\boldsymbol{\lambda^{n} x . e_1}$ in **e** carries an annotation **n** ∈ **{1, ω}** to indicate how often the lambda is called.

$$\epsilon \mapsto \texttt{let } f = \lambda x.\lambda y.\ x \texttt{ True in } f\ p\ q \ \downarrow C^1(U)$$

$$\Rightarrow \langle \bullet; \{p \mapsto 1 * C^1(U), q \mapsto A\}\rangle$$

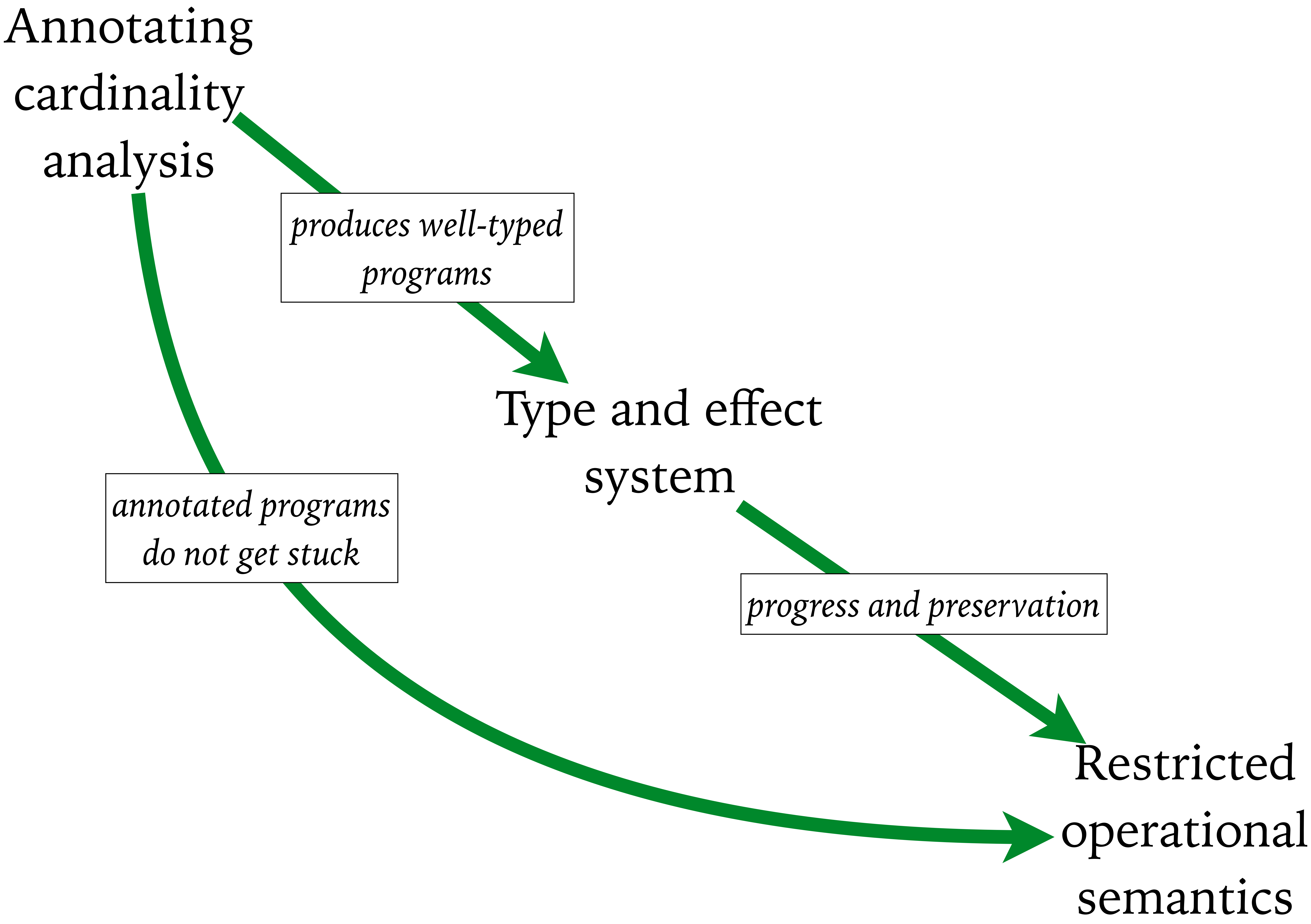
$$\rightsquigarrow$$

$$\texttt{let } f \overset{1}{=} \lambda^1 x.\lambda^1 y.\ x \texttt{ True in } f\ p\ q$$

Soundness

Restricted operational semantics

(makes sure that the annotations are respected)


Annotating cardinality analysis
produces well-typed programs
Type and effect system
annotated programs do not get stuck
progress and preservation
Restricted operational semantics

Small-Step CBN Machine

Sestoft:JFP97

$$e_1$$

Small-Step CBN Machine

$$\langle H_1, e_1, S_1 \rangle$$

Small-Step CBN Machine

$$\langle H_1, e_1, S_1 \rangle \longrightarrow \quad \ldots \quad \longrightarrow \langle H_n, e_n, S_n \rangle$$

$$e_1 \rightsquigarrow \mathsf{e}_1$$

Erasing Annotations

$$\mathrm{e}_1^{\natural} = e_1$$

$$\langle \mathsf{H}_1, \mathsf{e}_1, \mathsf{S}_1 \rangle$$

Restricted CBN Machine

$$\langle \mathsf{H}_1, \mathsf{e}_1, \mathsf{S}_1 \rangle \hookrightarrow \;\ldots\; \hookrightarrow \langle \mathsf{H}_n, \mathsf{e}_n, \mathsf{S}_n \rangle$$

- *1*-annotated lambdas can be called at most once;
- *1*-annotated bindings can be used only once;
- *0*-annotated bindings cannot be used at all.

Soundness Theorem

An analysis-annotated program
behaves *the same way* under restricted semantics
as the original program
under the normal semantics.

Soundness Theorem

If $\epsilon \mathrel{|\!\!\blacktriangleright} e_1 \downarrow U \Rightarrow \langle \tau, \epsilon \rangle \rightsquigarrow \mathsf{e}_1$

and $\langle \epsilon \,;\, e_1 \,;\, \epsilon \rangle \longrightarrow^{k} \langle H \,;\, e_2 \,;\, S \rangle$

then

$$\langle \epsilon \,;\, \mathsf{e}_1 \,;\, \epsilon \rangle \hookrightarrow^{k}_{\exists} \langle \mathsf{H} \,;\, \mathsf{e}_2 \,;\, \mathsf{S} \rangle$$

such that $\langle \mathsf{H}^{\natural}, \mathsf{e}_2^{\natural}, \mathsf{S}^{\natural} \rangle = \langle H, e_2, S \rangle$

Cardinality-Enabled Optimisations

1. Let-in floating optimisation

$$\mathtt{let}\ z \stackrel{m_1}{=} \mathtt{e}_1\ \mathtt{in}\ (\mathtt{let}\ f \stackrel{m_2}{=} \lambda^1 x\ .\ \mathtt{e}\ \mathtt{in}\ \mathtt{e}_2)$$

$$\texttt{let}\ z \overset{m_1}{=} \texttt{e}_1\ \texttt{in}\ (\texttt{let}\ f \overset{m_2}{=} \lambda^{1} x\ .\ \texttt{e}\ \texttt{in}\ \texttt{e}_2)$$

$$\Longrightarrow \quad \texttt{let}\ f \overset{m_2}{=} \lambda^{1} x\ .\ (\texttt{let}\ z \overset{m_1}{=} \texttt{e}_1\ \texttt{in}\ \texttt{e})\ \texttt{in}\ \texttt{e}_2,$$

for any m_1, m_2 and $z \notin FV(\texttt{e}_2)$.

Improvement Theorem 1

Let-in floating
does not increase the number
of execution steps.

Improvement Theorem 1

For any H and S, if

$$\langle \mathsf{H} ; \texttt{let}\ z \overset{m}{=} \mathsf{e}_1\ \texttt{in}\ (\texttt{let}\ f \overset{m_1}{=} \lambda^1 x\ .\ \mathsf{e}\ \texttt{in}\ \mathsf{e}_2) ; \mathsf{S}\rangle \Downarrow^{N}$$

and $z \notin FV(\mathsf{e}_2)$

then

$$\langle \mathsf{H} ; \texttt{let}\ f \overset{m_1}{=} \lambda^1 x\ .\ (\texttt{let}\ z \overset{m}{=} \mathsf{e}_1\ \texttt{in}\ \mathsf{e})\ \texttt{in}\ \mathsf{e}_2 ; \mathsf{S}\rangle \Downarrow^{\leq N}$$

where $\sigma \Downarrow^N$ means "terminates in N steps".

2. Smart Execution

$$e_1$$

Optimised CBN Machine

$$\langle \mathsf{H}_1, \mathsf{e}_1, \mathsf{S}_1 \rangle \Longrightarrow \ \ldots \ \Longrightarrow \langle \mathsf{H}_n, \mathsf{e}_n, \mathsf{S}_n \rangle$$

- ***1***-annotated bindings are *not* memoised;
- ***0***-annotated bindings are *skipped*.

Improvement Theorem 2

Optimising semantics
works *faster* on elaborated expressions
and produces coherent results.

Improvement Theorem 2

If $\epsilon \vdash\!\!\blacktriangleright e_1 \downarrow U \Rightarrow \langle \tau, \epsilon \rangle \rightsquigarrow \mathsf{e}_1$

and $\langle \epsilon \,;\, e_1 \,;\, \epsilon \rangle \xrightarrow{k} \boxed{\langle H \,;\, e_2 \,;\, S \rangle}$

then

$$\langle \epsilon, \mathsf{e}_1, \epsilon \rangle \stackrel{m}{\Longrightarrow} \boxed{\langle gc(\mathsf{H}), \mathsf{e}_2, gc(\mathsf{S}) \rangle}$$

such that $m \leq k$

and $\boxed{\langle \mathsf{H}^\natural, \mathsf{e}_2^\natural, \mathsf{S}^\natural \rangle} = \langle H, e_2, S \rangle$

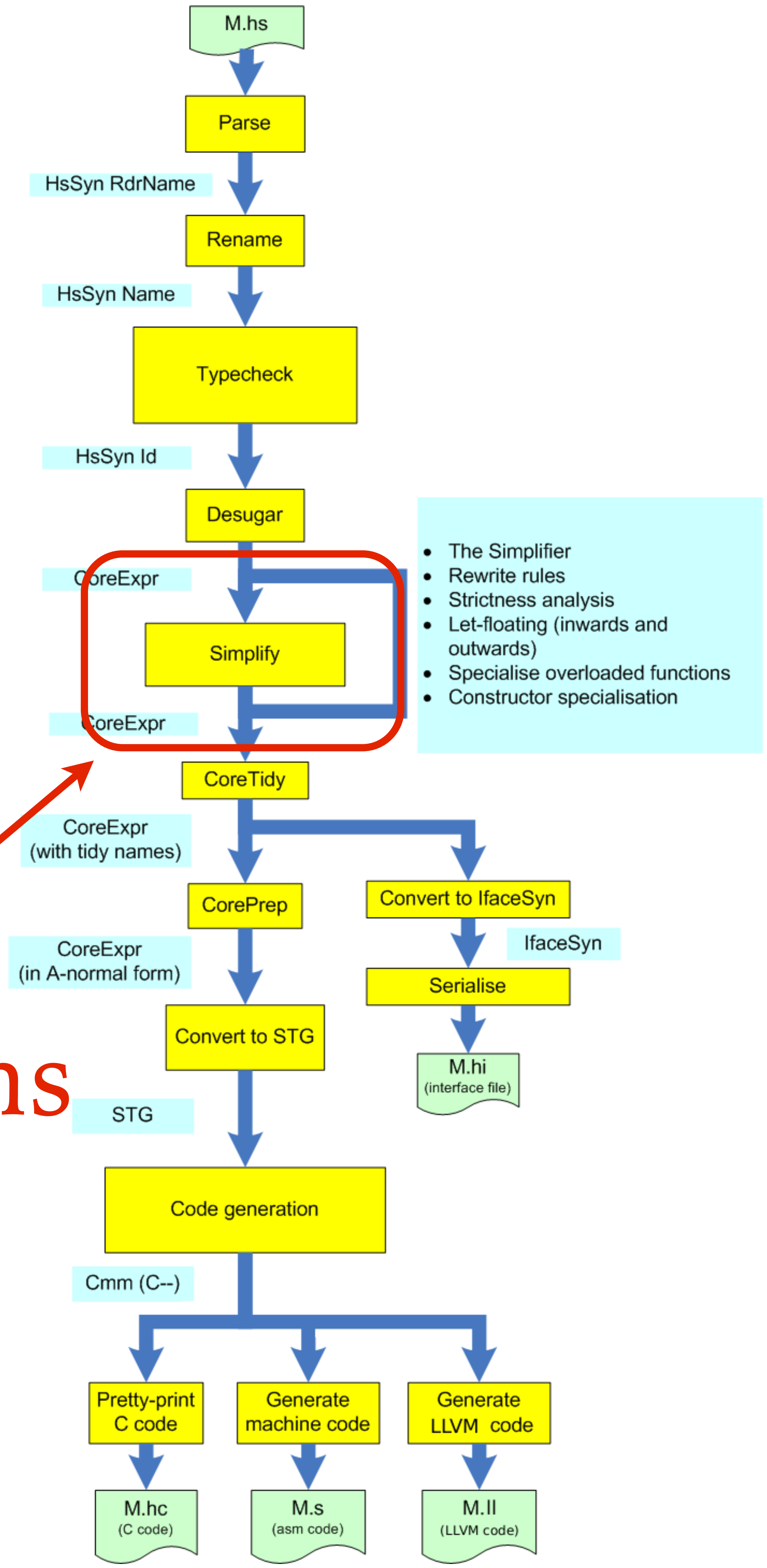
Implementation
and
Evaluation

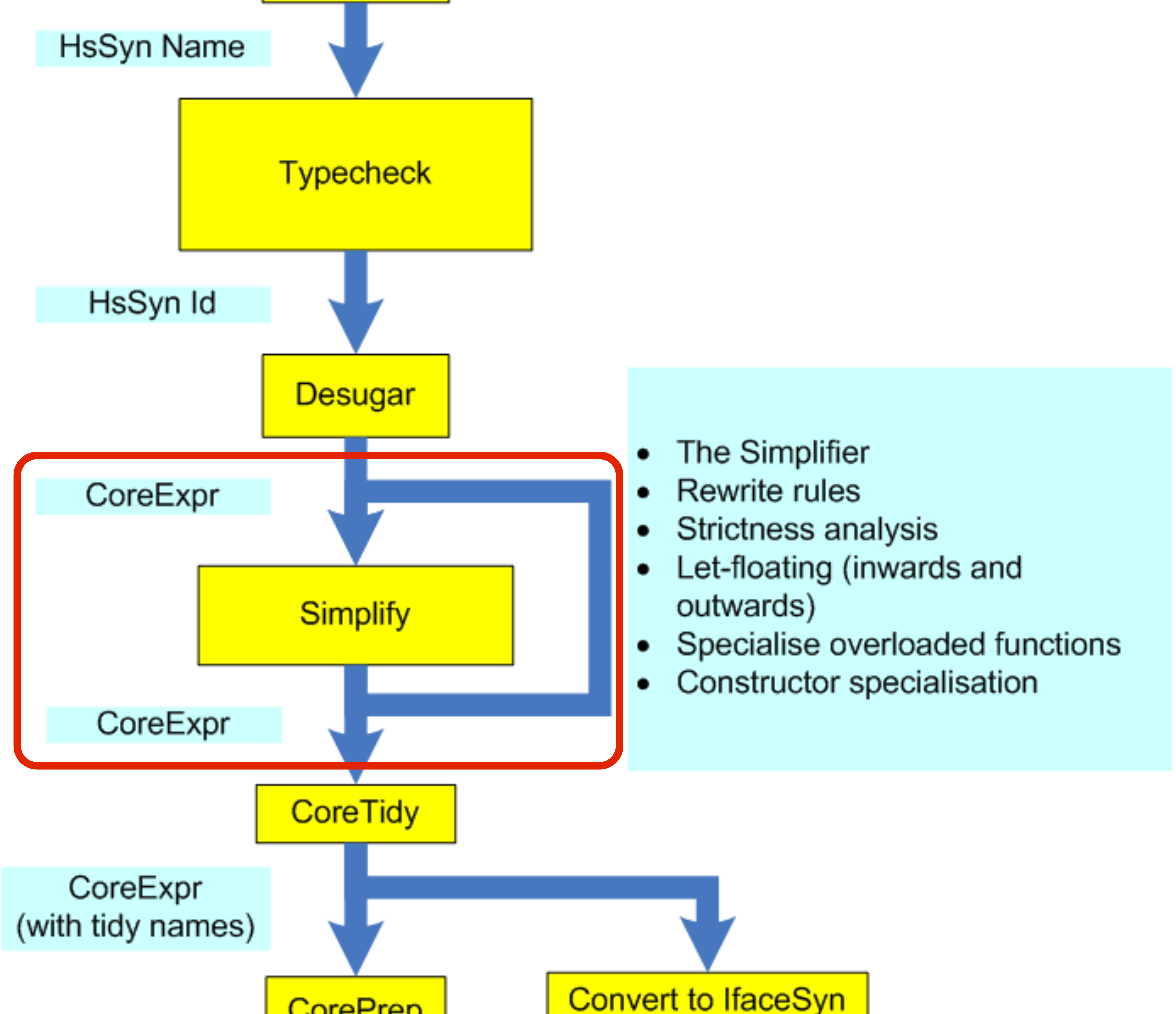
GHC Compilation Pipeline

A number of Intermediate Languages

- Haskell Source
- Core
- Spineless Tagless G-Machine
- C--
- C / Machine Code / LLVM Code

Most of interesting optimizations happen here




HsSyn Name
Typecheck
HsSyn Id
Desugar
CoreExpr
Simplify
CoreExpr
CoreTidy
CoreExpr
(with tidy names)
CorePrep
Convert to IfaceSyn
• The Simplifier
• Rewrite rules
• Strictness analysis
• Let-floating (inwards and outwards)
• Specialise overloaded functions
• Constructor specialisation

GHC Core

- A tiny language, to which Haskell sources are de-sugared;
- Based on explicitly typed System F with type equality coercions;
- Used as a base platform for analyses and optimisations;
- All names are fully-qualified;
- **if-then-else** is compiled to **case**-expressions;
- Variables have additional metadata;
- Type class constraints are compiled into record parameters.

Core Syntax

```
data Expr b
  = Var   Id
  | Lit   Literal
  | App   (Expr b) (Expr b)
  | Lam   b (Expr b)
  | Let   (Bind b) (Expr b)
  | Case  (Expr b) b Type [Alt b]
  | Cast  (Expr b) Coercion
  | Tick  (Tickish Id) (Expr b)
  | Type  Type
  | Coercion Coercion

data Bind b = NonRec b (Expr b)
            | Rec [(b, (Expr b))]

type Alt b = (AltCon, [b], Expr b)

data AltCon
  = DataAlt DataCon
  | LitAlt  Literal
  | DEFAULT
```

How to See Core

Desugared Core

```
> ghc -ddump-ds Program.hs
```

Core with Strictness Annotations

```
> ghc -O2 -ddump-stranal Program.hs
```

Core after Worker/Wrapper Split

```
> ghc -O2 -ddump-worker-wrapper Program.hs
```

Try it on

```
module Program where

squash f = f 42

costly x = product [1..x]

foo xs = squash (\n -> sum (map (+ n) (map costly xs)))
```

- The analysis and optimisations are implemented in Glasgow Haskell Compiler (GHC v7.8 and newer): http://github.com/ghc/ghc
- Added 250 LOC to 140 KLOC compiler;
- Runs simultaneously with the strictness analyser;
- Evaluated on
  - **nofib** benchmark suite,
  - various **hackage** libraries,
  - the Benchmark Game programs,
  - GHC itself.

Results on nofib

Program	Synt. λ^1	Synt. Thnk[1]	RT Thnk[1] *
`anna`	4.0%	7.2%	2.9%
`bspt`	5.0%	15.4%	1.5%
`cacheprof`	7.6%	11.9%	5.1%
`calendar`	5.7%	0.0%	0.2%
`constraints`	2.0%	3.2%	4.5%
... and 72 more programs			
Arithmetic mean	10.3%	12.6%	5.5%

* as linked and run with libraries

Results on nofib

Program	Allocs		Runtime	
	No hack	Hack	No hack	Hack
`anna`	-2.1%	-0.2%	+0.1%	-0.0%
`bspt`	-2.2%	-0.0%	-0.0%	+0.0%
`cacheprof`	-7.9%	-0.6%	-6.1%	-5.0%
`calendar`	-9.2%	+0.2%	-0.0%	-0.0%
`constraints`	-0.9%	-0.0%	-1.2%	-0.2%
... and 72 more programs				
Min	-95.5%	-10.9%	-28.2%	-12.1%
Max	+3.5%	+0.5%	+1.8%	+2.8%
Geometric mean	-6.0%	-0.3%	-2.2%	-1.4%

The hack (due to A. Gill): hardcode argument cardinalities for `build`, `foldr` and `runST`.

Compiling with optimised GHC

- We compiled GHC *itself* with cardinality optimisations;
- Then we measured improvement in *compilation runtimes*.

Program	LOC	GHC Alloc Δ		GHC RT Δ	
		No hack	Hack	No hack	Hack
`anna`	5740	-1.6%	-1.5%	-0.8%	-0.4%
`cacheprof`	1600	-1.7%	-0.4%	-2.3%	-1.8%
`fluid`	1579	-1.9%	-1.9%	-2.8%	-1.6%
`gamteb`	1933	-0.5%	-0.1%	-0.5%	-0.1%
`parser`	2379	-0.7%	-0.2%	-2.6%	-0.6%
`veritas`	4674	-1.4%	-0.3%	-4.5%	-4.1%

Beyond GHC optimisations

Program Synthesis by Type-Guided Abstraction Refinement

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RANJIT JHALA, UC San Diego, USA
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- Hoogle+ synthesis algorithm (POPL'20) relies on cardinality analysis to eliminate terms where some of the inputs are unused.

To take away

- **Cardinality analysis** is simple to design and understand:
  it's all about *usage demands* and *demand transformers;*
- It is cheap to implement: we added only 250 LOC to GHC;
- It is conservative, which makes it fast and modular;
- *Call demands* make it higher-order, so the analysis can infer demands on higher-order function arguments;
- It is reasonably efficient: optimised GHC compiles up to ***4%*** faster.
- The ideas of cardinality analysis extend beyond just optimisations in GHC.

Thanks!