Deductive Synthesis of Programs that Alter Data Structures

Nadia Polikarpova
University of California San Diego

Ilya Sergey
UCL

Let’s *swap* values of two *distinct* pointers

Let's *swap* values of two *distinct* pointers

Let's *swap* values of two *distinct* pointers

swap

```
void swap(loc x, loc y)
```

$$\{\ \mathtt{x} \mapsto a \land \mathtt{y} \mapsto b\ \}$$

```
void swap(loc x, loc y)
```

$$\{\ x \mapsto a \wedge y \mapsto b\ \}$$

```
void swap(loc x, loc y)
```

$$\{\ x \mapsto b \wedge y \mapsto a\ \}$$

“separately”

$$\{\ x \mapsto a * y \mapsto b\ \}$$

```
void swap(loc x, loc y)
```

$$\{\ x \mapsto b * y \mapsto a\ \}$$

Peter W. O'Hearn, John C. Reynolds, Hongseok Yang:
Local Reasoning about Programs that Alter Data Structures. CSL 2001

$$\{ x \mapsto a * y \mapsto b \}$$

```
void swap(loc x, loc y)
```

$$\{ x \mapsto b * y \mapsto a \}$$

$$\{\ x \mapsto \boxed{a} * y \mapsto \boxed{b}\ \}$$

```
void swap(loc x, loc y)
```

$$\{\ x \mapsto \boxed{b} * y \mapsto \boxed{a}\ \}$$

$$\{\ x \mapsto \boxed{a} * y \mapsto b\ \}$$

??

$$\{\ x \mapsto b * y \mapsto \boxed{a}\ \}$$

```
let a2 = *x;
```

$$\{\ x \mapsto a2 * y \mapsto \boxed{b}\ \}$$

??

$$\{\ x \mapsto \boxed{b} * y \mapsto a2\ \}$$

```
let a2 = *x;
let b2 = *y;
```

$\{\ x \mapsto a2 * y \mapsto b2\ \}$

??

$\{\ x \mapsto b2 * y \mapsto a2\ \}$

```
let a2 = *x;
let b2 = *y;
*x = b2;
```

$$\{\ x \mapsto b2 * y \mapsto b2\ \}$$

??

$$\{\ x \mapsto b2 * y \mapsto a2\ \}$$

```
let a2 = *x;
let b2 = *y;
*x = b2;
*y = a2;
```

$$\{\ x \mapsto b2 \ast y \mapsto a2\ \}$$

??

$$\{\ x \mapsto b2 \ast y \mapsto a2\ \}$$

```
let a2 = *x;
let b2 = *y;
*x = b2;
*y = a2;
```

$$\{\ x \mapsto b2 * y \mapsto a2\ \}$$

??

$$\{\ x \mapsto b2 * y \mapsto a2\ \}$$

$$x \mapsto b2 * y \mapsto a2 \ \vdash \ x \mapsto b2 * y \mapsto a2$$

```
let a2 = *x;
let b2 = *y;
*x = b2;
*y = a2;
```

$$\{\ x \mapsto b2 \ast y \mapsto a2\ \}$$

??

$$\{\ x \mapsto b2 \ast y \mapsto a2\ \}$$

$$x \mapsto b2 \ast y \mapsto a2 \ \vdash\ x \mapsto b2 \ast y \mapsto a2$$

```
void swap(loc x, loc y) {
        let a2 = *x;
        let b2 = *y;
        *x = b2;
        *y = a2;
}
```

Reasoning with Symbolic Heaps

Symbolic Heap Entailment

$$P \vdash Q$$

Any heap (state) that satisfies **P**, also satisfies **Q**.

Program Validity *wrt.* Pre/Postcondition

$$\{ P \} \quad c \quad \{ Q \}$$

If the initial state satisfies **P**, then, after **c** terminates, the final state satisfies **Q**.

Transforming Entailment

(this work)

$$P \rightsquigarrow Q$$

There *exists* a program **c**, such that
for any initial state satisfying P,
c, after it terminates,
will transform to a state satisfying Q.

$P \vdash Q$ implies $P \rightsquigarrow Q$

“Proof”: `skip`

$$x \mapsto a \quad \rightsquigarrow \quad x \mapsto 42$$

"Proof": `*x = 42`

$x \mapsto a \rightsquigarrow x \mapsto 42$ | `*x = 42`

$$P \rightsquigarrow Q \mid \mathbf{c}$$

P transforms to Q via a program **c**.

Theorem:

$$P \rightsquigarrow Q \mid c \quad \text{implies} \quad \{\, P \,\}\ c\ \{\, Q \,\}$$

$\{P\}$?? $\{Q\}$ vs $P \rightsquigarrow Q \mid c$

Declarative *Algorithmic*

Synthetic Separation Logic

$$\Gamma \,;\, P \rightsquigarrow Q \mid c$$

$$\Gamma \; ; \; P \rightsquigarrow Q \mid c$$

- (Γ, P, Q) — *goal*
- **GV** (Γ, P, Q) — *ghost* variables (scope: *pre/postcondition*)
- **EV** (Γ, P, Q) — *existentials* (scope: *postcondition*)

$$\Gamma; \{\mathsf{emp}\} \rightsquigarrow \{\mathsf{emp}\} \mid ??$$

$$\Gamma; \{\mathsf{emp}\} \rightsquigarrow \{\mathsf{emp}\} \mid \mathbf{\texttt{skip}} \qquad (\mathrm{Emp})$$

$$a \in \mathrm{GV}(\Gamma, P, Q)$$

$$\Gamma; \{\, x \mapsto a * P \,\} \rightsquigarrow \{\, Q \,\} \mid ??$$

$$\frac{a \in \mathrm{GV}(\Gamma, P, Q) \qquad y \text{ is fresh} \qquad \Gamma, y\,;\, [y/a]\{\, x \mapsto y * P \,\} \rightsquigarrow [y/a]\{\, Q \,\} \mid \texttt{c}}{\Gamma;\, \{\, x \mapsto a * P \,\} \rightsquigarrow \{\, Q \,\} \mid \texttt{let y = *x; c}}\ \text{(Read)}$$

$$\Gamma; \{ x \mapsto - * P \} \rightsquigarrow \{ x \mapsto e * Q \} \mid ??$$

$$\frac{\mathit{Vars}(e) \subseteq \Gamma \qquad \Gamma ; \{ x \mapsto e * P \} \rightsquigarrow \{ x \mapsto e * Q \} \mid \texttt{c}}{\Gamma ; \{ x \mapsto - * P \} \rightsquigarrow \{ x \mapsto e * Q \} \mid \texttt{*x = e; c}} \ \text{(Write)}$$

$$\Gamma; \{ P * R \} \rightsquigarrow \{ Q * R \} \mid ??$$

$$\frac{\mathrm{EV}(\Gamma, P, Q) \cap \mathit{Vars}(R) = \emptyset \qquad \Gamma ; \{\, P \,\} \rightsquigarrow \{\, Q \,\} \mid \mathbf{c}}{\Gamma ; \{\, P * R \,\} \rightsquigarrow \{\, Q * R \,\} \mid \mathbf{c}} \; \text{(Frame)}$$

$$\Gamma; \{\mathsf{emp}\} \rightsquigarrow \{\mathsf{emp}\} \mid \texttt{skip} \qquad \text{(Emp)}$$

$$\frac{a \in \mathrm{GV}(\Gamma, P, Q) \qquad y \text{ is fresh} \qquad \Gamma, y ; [y/a]\{ x \mapsto y * P \} \rightsquigarrow [y/a]\{ Q \} \mid \texttt{c}}{\Gamma; \{ x \mapsto a * P \} \rightsquigarrow \{ Q \} \mid \texttt{let y = *x; c}} \text{(Read)}$$

$$\frac{\mathrm{EV}(\Gamma, P, Q) \cap \mathit{Vars}(R) = \emptyset \qquad \Gamma ; \{ P \} \rightsquigarrow \{ Q \} \mid \texttt{c}}{\Gamma; \{ P * R \} \rightsquigarrow \{ Q * R \} \mid \texttt{c}} \text{(Frame)}$$

$$\frac{\mathit{Vars}(e) \subseteq \Gamma \qquad \Gamma ; \{ x \mapsto e * P \} \rightsquigarrow \{ x \mapsto e * Q \} \mid \texttt{c}}{\Gamma; \{ x \mapsto - * P \} \rightsquigarrow \{ x \mapsto e * Q \} \mid \texttt{*x = e; c}} \text{(Write)}$$

$$\{ x \mapsto a * y \mapsto b \}$$

```
void swap(loc x, loc y)
```

$$\{ x \mapsto b * y \mapsto a \}$$

$$\{\, x, y \,\} \,;\, \{\, x \mapsto a * y \mapsto b \,\} \rightsquigarrow \{\, x \mapsto b * y \mapsto a \,\} \quad | \quad \text{??}$$

$$\frac{\{\, \mathsf{x}, \mathsf{y}, \mathsf{a2} \,\} ;\ \{\, \mathsf{x} \mapsto \mathsf{a2} * \mathsf{y} \mapsto b \,\} \rightsquigarrow \{\, \mathsf{x} \mapsto b * \mathsf{y} \mapsto \mathsf{a2} \,\} \mid\ \texttt{??}}{\{\, \mathsf{x}, \mathsf{y} \,\} ;\ \{\, \mathsf{x} \mapsto a * \mathsf{y} \mapsto b \,\} \rightsquigarrow \{\, \mathsf{x} \mapsto b * \mathsf{y} \mapsto a \,\} \mid\ \texttt{let a2 = *x; ??}} \text{(Read)}$$

$$\dfrac{\dfrac{\{\, x, y, a2, b2 \,\} ; \{\, x \mapsto a2 * y \mapsto b2 \,\} \leadsto \{\, x \mapsto b2 * y \mapsto a2 \,\} \mid \texttt{??}}{\{\, x, y, a2 \,\} ; \{\, x \mapsto a2 * y \mapsto b \,\} \leadsto \{\, x \mapsto b * y \mapsto a2 \,\} \mid \texttt{let b2 = *y; ??}}\ \text{(Read)}}{\{\, x, y \,\} ; \{\, x \mapsto a * y \mapsto b \,\} \leadsto \{\, x \mapsto b * y \mapsto a \,\} \mid \texttt{let a2 = *x; ??}}\ \text{(Read)}$$

$$\{\, \mathrm{x}, \mathrm{y}, \mathrm{a2}, \mathrm{b2} \,\} \,;\, \{\, \mathrm{x} \mapsto \mathrm{b2} * \mathrm{y} \mapsto \mathrm{b2} \,\} \rightsquigarrow \{\, \mathrm{x} \mapsto \mathrm{b2} * \mathrm{y} \mapsto \mathrm{a2} \,\} \mid \texttt{??}$$

(Write)

$$\{\, \mathrm{x}, \mathrm{y}, \mathrm{a2}, \mathrm{b2} \,\} \,;\, \{\, \mathrm{x} \mapsto \mathrm{a2} * \mathrm{y} \mapsto \mathrm{b2} \,\} \rightsquigarrow \{\, \mathrm{x} \mapsto \mathrm{b2} * \mathrm{y} \mapsto \mathrm{a2} \,\} \mid \texttt{*x = b2; ??}$$

(Read)

$$\{\, \mathrm{x}, \mathrm{y}, \mathrm{a2} \,\} \,;\, \{\, \mathrm{x} \mapsto \mathrm{a2} * \mathrm{y} \mapsto b \,\} \rightsquigarrow \{\, \mathrm{x} \mapsto b * \mathrm{y} \mapsto \mathrm{a2} \,\} \mid \texttt{let b2 = *y; ??}$$

(Read)

$$\{\, \mathrm{x}, \mathrm{y} \,\} \,;\, \{\, \mathrm{x} \mapsto a * \mathrm{y} \mapsto b \,\} \rightsquigarrow \{\, \mathrm{x} \mapsto b * \mathrm{y} \mapsto a \,\} \mid \texttt{let a2 = *x; ??}$$

$$\{\, x, y, a2, b2 \,\} ; \{\, y \mapsto b2 \,\} \rightsquigarrow \{\, y \mapsto a2 \,\} \mid \texttt{??}$$

(Frame)

$$\{\, x, y, a2, b2 \,\} ; \{\, x \mapsto b2 * y \mapsto b2 \,\} \rightsquigarrow \{\, x \mapsto b2 * y \mapsto a2 \,\} \mid \texttt{??}$$

(Write)

$$\{\, x, y, a2, b2 \,\} ; \{\, x \mapsto a2 * y \mapsto b2 \,\} \rightsquigarrow \{\, x \mapsto b2 * y \mapsto a2 \,\} \mid \texttt{*x = b2; ??}$$

(Read)

$$\{\, x, y, a2 \,\} ; \{\, x \mapsto a2 * y \mapsto b \,\} \rightsquigarrow \{\, x \mapsto b * y \mapsto a2 \,\} \mid \texttt{let b2 = *y; ??}$$

(Read)

$$\{\, x, y \,\} ; \{\, x \mapsto a * y \mapsto b \,\} \rightsquigarrow \{\, x \mapsto b * y \mapsto a \,\} \mid \texttt{let a2 = *x; ??}$$

$$\{\, x, y, a2, b2 \,\} ; \{\, y \mapsto a2 \,\} \leadsto \{\, y \mapsto a2 \,\} \mid \texttt{??}$$

(Write)

$$\{\, x, y, a2, b2 \,\} ; \{\, y \mapsto b2 \,\} \leadsto \{\, y \mapsto a2 \,\} \mid \texttt{*y = a2; ??}$$

(Frame)

$$\{\, x, y, a2, b2 \,\} ; \{\, x \mapsto b2 * y \mapsto b2 \,\} \leadsto \{\, x \mapsto b2 * y \mapsto a2 \,\} \mid \texttt{??}$$

(Write)

$$\{\, x, y, a2, b2 \,\} ; \{\, x \mapsto a2 * y \mapsto b2 \,\} \leadsto \{\, x \mapsto b2 * y \mapsto a2 \,\} \mid \texttt{*x = b2; ??}$$

(Read)

$$\{\, x, y, a2 \,\} ; \{\, x \mapsto a2 * y \mapsto b \,\} \leadsto \{\, x \mapsto b * y \mapsto a2 \,\} \mid \texttt{let b2 = *y; ??}$$

(Read)

$$\{\, x, y \,\} ; \{\, x \mapsto a * y \mapsto b \,\} \leadsto \{\, x \mapsto b * y \mapsto a \,\} \mid \texttt{let a2 = *x; ??}$$

$$\{\, x, y, a2, b2 \,\} ;\ \{\ \mathsf{emp}\ \} \leadsto \{\ \mathsf{emp}\ \} \mid \texttt{??}$$

(Frame)

$$\{\, x, y, a2, b2 \,\} ;\ \{\ y \mapsto a2\ \} \leadsto \{\ y \mapsto a2\ \} \mid \texttt{??}$$

(Write)

$$\{\, x, y, a2, b2 \,\} ;\ \{\ y \mapsto b2\ \} \leadsto \{\ y \mapsto a2\ \} \mid \texttt{*y = a2; ??}$$

(Frame)

$$\{\, x, y, a2, b2 \,\} ;\ \{\ x \mapsto b2 * y \mapsto b2\ \} \leadsto \{\ x \mapsto b2 * y \mapsto a2\ \} \mid \texttt{??}$$

(Write)

$$\{\, x, y, a2, b2 \,\} ;\ \{\ x \mapsto a2 * y \mapsto b2\ \} \leadsto \{\ x \mapsto b2 * y \mapsto a2\ \} \mid \texttt{*x = b2; ??}$$

(Read)

$$\{\, x, y, a2 \,\} ;\ \{\ x \mapsto a2 * y \mapsto b\ \} \leadsto \{\ x \mapsto b * y \mapsto a2\ \} \mid \texttt{let b2 = *y; ??}$$

(Read)

$$\{\, x, y \,\} ;\ \{\ x \mapsto a * y \mapsto b\ \} \leadsto \{\ x \mapsto b * y \mapsto a\ \} \mid \texttt{let a2 = *x; ??}$$

(Emp)

$$\{ x, y, a2, b2 \} ; \{ \mathsf{emp} \} \rightsquigarrow \{ \mathsf{emp} \} \mid \texttt{skip}$$

(Frame)

$$\{ x, y, a2, b2 \} ; \{ y \mapsto a2 \} \rightsquigarrow \{ y \mapsto a2 \} \mid \texttt{??}$$

(Write)

$$\{ x, y, a2, b2 \} ; \{ y \mapsto b2 \} \rightsquigarrow \{ y \mapsto a2 \} \mid \boxed{\texttt{*y = a2;}} \; \texttt{??}$$

(Frame)

$$\{ x, y, a2, b2 \} ; \{ x \mapsto b2 * y \mapsto b2 \} \rightsquigarrow \{ x \mapsto b2 * y \mapsto a2 \} \mid \texttt{??}$$

(Write)

$$\{ x, y, a2, b2 \} ; \{ x \mapsto a2 * y \mapsto b2 \} \rightsquigarrow \{ x \mapsto b2 * y \mapsto a2 \} \mid \boxed{\texttt{*x = b2;}} \; \texttt{??}$$

(Read)

$$\{ x, y, a2 \} ; \{ x \mapsto a2 * y \mapsto b \} \rightsquigarrow \{ x \mapsto b * y \mapsto a2 \} \mid \boxed{\texttt{let b2 = *y;}} \; \texttt{??}$$

(Read)

$$\{ x, y \} ; \{ x \mapsto a * y \mapsto b \} \rightsquigarrow \{ x \mapsto b * y \mapsto a \} \mid \boxed{\texttt{let a2 = *x;}} \; \texttt{??}$$

```
void swap(loc x, loc y) {
    let a2 = *x;
    let b2 = *y;
    *x = b2;
    *y = a2;
}
```

Unification and Non-Determinism

$$\{ x \mapsto 239 * y \mapsto 30 \}$$

```
void pick(loc x, loc y)
```

$$\{ x \mapsto z * y \mapsto z \}$$

$$\{ x \mapsto 239 * y \mapsto 30 \}$$

```
void pick(loc x, loc y)
```

$$\{ x \mapsto z * y \mapsto z \}$$

$$\frac{[\sigma]R' = R \quad \emptyset \neq \mathrm{dom}(\sigma) \subseteq EV(\Gamma, P, Q) \quad \Gamma; \{ P * R \} \rightsquigarrow [\sigma]\{ Q * R' \} \mid c}{\Gamma; \{ P * R \} \rightsquigarrow \{ Q * R' \} \mid c} \text{(UnifyHeaps)}$$

$$\{x, y\};\ \{x \mapsto 239 * y \mapsto 30\} \rightsquigarrow \{x \mapsto z * y \mapsto z\}$$

$R = x \mapsto 239$

$R' = x \mapsto z$

$\sigma = [z \mapsto 239]$

```
void pick(loc x, loc y) {
    *y = 239;
}
```

$R = y \mapsto 30$

$R' = y \mapsto z$

$\sigma = [z \mapsto 30]$

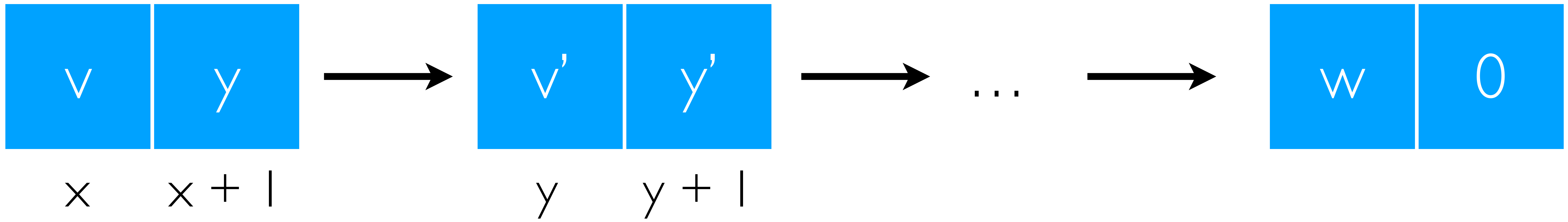
```
void pick(loc x, loc y) {
    *x = 30;
}
```

Pure Parts

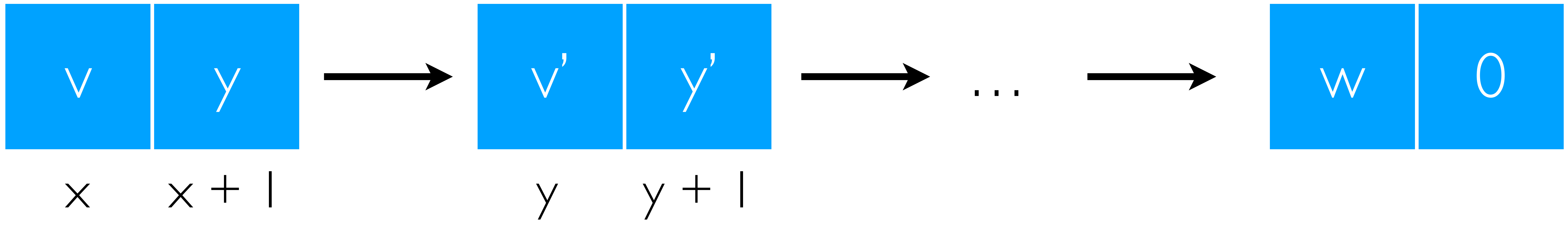
$$\Gamma \,;\, \{P\} \rightsquigarrow \{Q\} \mid c$$

$$\Gamma \,;\, \{\, \varphi; P \,\} \rightsquigarrow \{\, \psi; Q \,\} \;\mid\; c$$

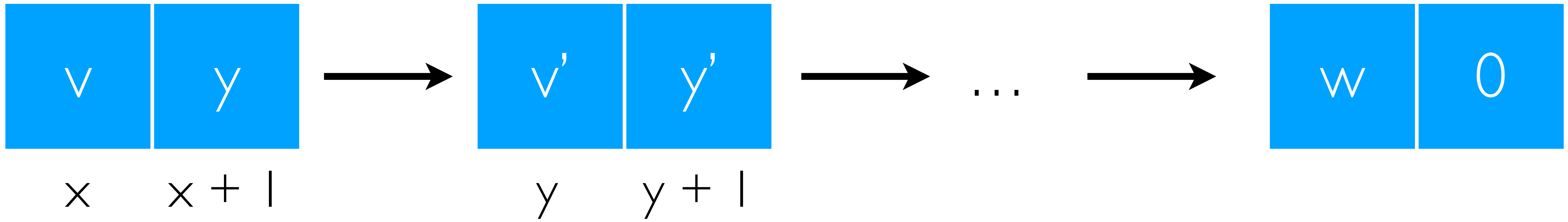
Inductive Predicates and Recursion


v
y
x
x + 1
v'
y'
y
y + 1
...
w
0

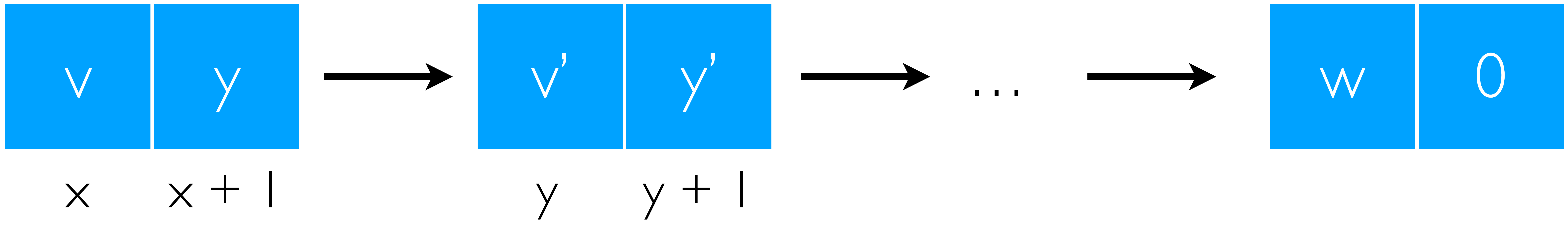
```
predicate lseg (loc x, set s) {
  | x = 0  ∧  { s = ∅          ;  emp }
  | x ≠ 0 ∧  { s = {v} ∪ s'   ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg(y, s') }
}
```



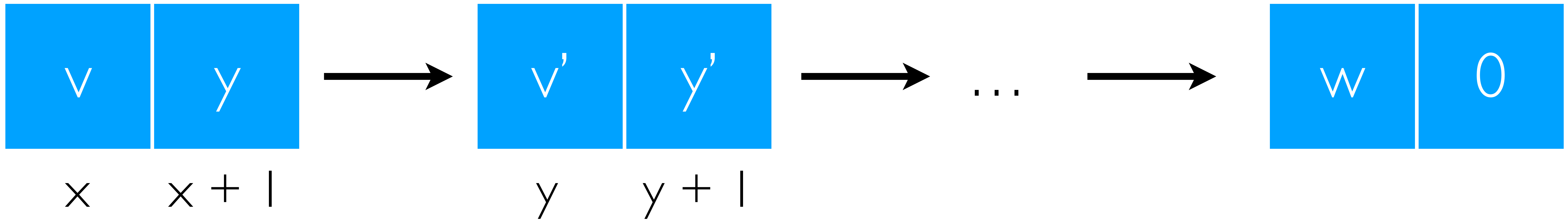
```
predicate lseg (loc x, set s) {
  | x = 0 ∧ { s = ∅              ; emp }
  | x ≠ 0 ∧ { s = {v} ∪ s'   ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg(y, s') }
}
```



$$
\begin{aligned}
&\textbf{predicate}\ \mathrm{lseg}\ (\textbf{loc}\ x,\ \textbf{set}\ s)\ \{\\
&\quad |\ x = 0\ \wedge\ \{\ s = \varnothing\ ;\ \mathrm{emp}\ \}\\
&\quad |\ x \neq 0\ \wedge\ \{\ s = \{v\} \cup s'\ ;\ [x, 2] * x \mapsto v * (x + 1) \mapsto y * \mathrm{lseg}(y, s')\ \}\\
&\}
\end{aligned}
$$



```
predicate lseg (loc x, set s) {
  | x = 0  ∧  { s = ∅         ;  emp }
  | x ≠ 0 ∧  { s = {v} ∪ s'   ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg(y, s') }
}
```

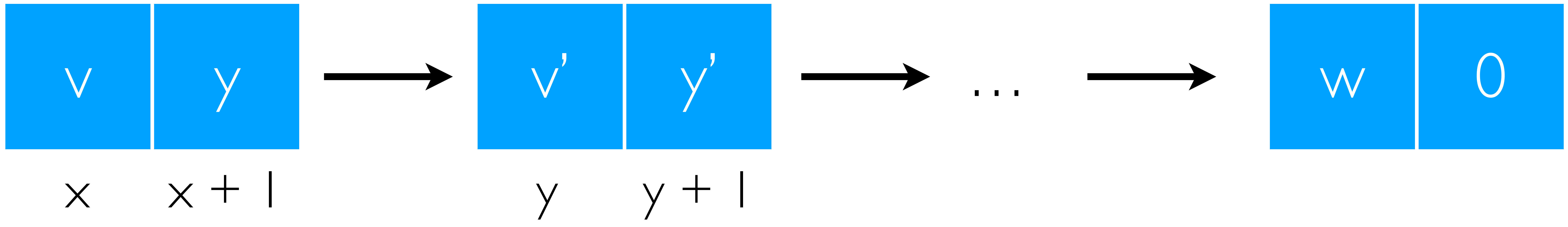


predicate lseg (**loc** x, **set** s) {

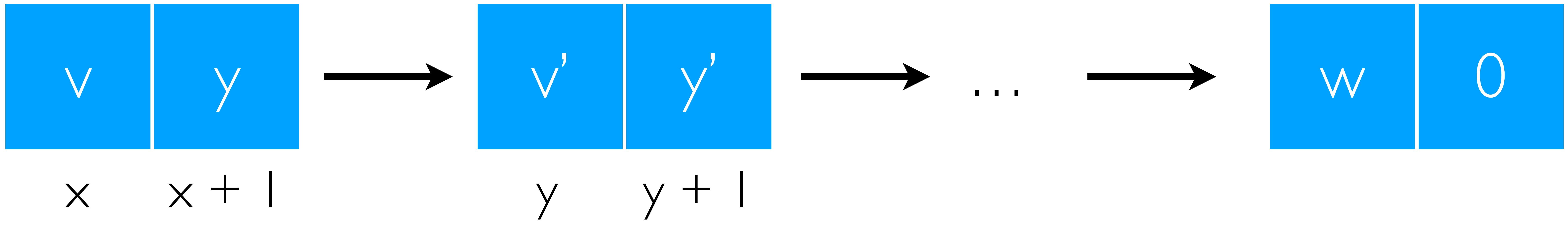
| $x = 0 \ \wedge \ \{ s = \emptyset \quad ; \ emp \}$

| $x \neq 0 \ \wedge \ \{ s = \{v\} \cup s' \quad ; [x, 2] * x \mapsto v * (x + 1) \mapsto y * lseg(y, s') \}$

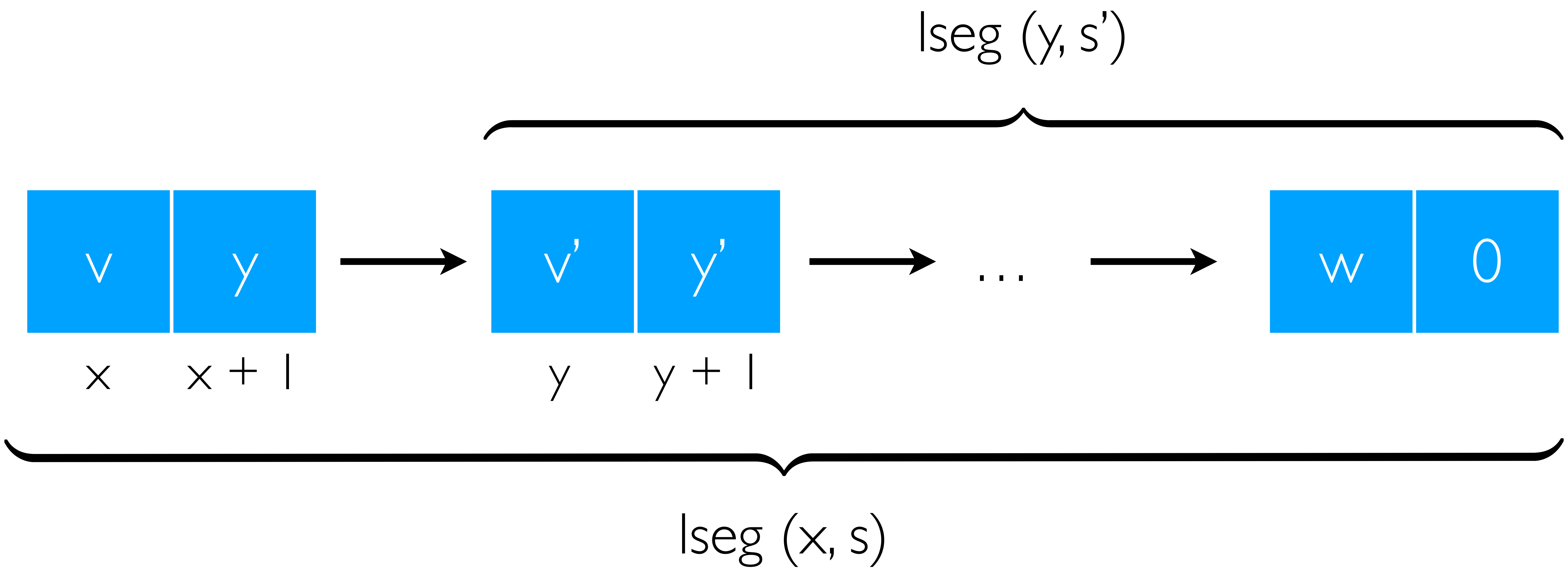
}



```
predicate lseg (loc x, set s) {
  | x = 0  ∧  { s = ∅          ;  emp }
  | x ≠ 0 ∧  { s = {v} ∪ s'   ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg(y, s') }
}
```



predicate lseg (**loc** x, **set** s) {
  | $x = 0 \;\wedge\; \{\, s = \varnothing \;;\; \mathrm{emp} \,\}$
  | $x \neq 0 \wedge \{\, s = \{v\} \cup s' \;;\; [x, 2] * x \mapsto v * (x + 1) \mapsto y * \mathrm{lseg}(y, s') \,\}$
}



```
predicate lseg (loc x, set s) {
  | x = 0  ∧  { s = ∅           ;  emp }
  | x ≠ 0 ∧  { s = {v} ∪ s'    ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg(y, s') }
}
```

{ lseg (x, s) }

```
void listfree(loc x)
```

{ emp }

predicate lseg (**loc** x, **set** s) {
  | $x = 0 \wedge \{ s = \varnothing \;;\; emp \}$
  | $x \neq 0 \wedge \{ s = \{v\} \cup s' \;;\; [x, 2] * x \mapsto v * (x + 1) \mapsto y * lseg(y, s') \}$
}

$\{ lseg^{1}(x, s) \}$ `void listfree(loc x)` $\{ emp \}$

$$\{ lseg^{0}(x, s) \}$$

??

$$\{ emp \}$$

predicate lseg (loc x, set s) {
| x = 0 ∧ { s = ∅ ; emp }
| x ≠ 0 ∧ { s = {v} ∪ s' ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg(y, s') }
}

{ $lseg^1$ (x, s) } `void listfree(loc x)` { emp }

{ $lseg^0$ (x, s) }

??

{ emp }

predicate lseg (loc x, set s) {
  | $x = 0 \land$ { $s = \emptyset$; emp }
  | $x \neq 0 \land$ { $s = \{v\} \cup s'$; $[x, 2] * x \mapsto v * (x + 1) \mapsto y *$ lseg(y, s') }
}

{ $lseg^1(x, s)$ } `void listfree(loc x)` { emp }

```
if (x == 0) {
```

{ $x = 0$; $lseg^0(x, s)$ }

```
    ??
```

{ emp }

```
} else {
```

{ $x \neq 0$; $lseg^0(x, s)$ }

```
    ??
```

{ emp }

```
}
```

```
predicate lseg (loc x, set s) {
  | x = 0 ∧ { s = ∅        ; emp }
  | x ≠ 0 ∧ { s = {v} ∪ s'  ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg(y, s') }
}
```

```
{ lseg¹ (x, s) } void listfree(loc x) { emp }

if (x == 0) {
  { x = 0 ∧ s = ∅ ; emp }

      ??

  { emp }

} else {
  { x ≠ 0 ∧ s = {v} ∪ s'  ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg¹ (y, s') }

      ??

  { emp }
}
```

predicate lseg (loc x, set s) {
| $x = 0 \wedge \{ s = \varnothing \; ; \; emp \}$
| $x \neq 0 \wedge \{ s = \{v\} \cup s' \; ; [x, 2] * x \mapsto v * (x + 1) \mapsto y * lseg(y, s') \}$
}

$\{ lseg^{1}(x, s) \}$ `void listfree(loc x)` $\{ emp \}$

```
if (x == 0) {
```

$\{ x = 0 \wedge s = \varnothing \; ; \; emp \}$

```
    skip
```

$\{ emp \}$

```
} else {
```

$\{ x \neq 0 \wedge s = \{v\} \cup s' \; ; [x, 2] * x \mapsto v * (x + 1) \mapsto y * lseg^{1}(y, s') \}$

```
    ??
```

$\{ emp \}$

```
}
```

predicate lseg (loc x, set s) {
  | x = 0 ∧ { s = ∅ ; emp }
  | x ≠ 0 ∧ { s = {v} ∪ s' ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg(y, s') }
}

{ $lseg^{1}$ (x, s) } `void listfree(loc x)` { emp }

```
if (x == 0) { } else {
```

{ x ≠ 0 ∧ s = {v} ∪ s' ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ $lseg^{1}$ (y, s') }

```
    ??
```

{ emp }

```
}
```

predicate lseg (loc x, set s) {
  | $x = 0 \wedge \{ s = \varnothing \; ; \; emp \}$
  | $x \neq 0 \wedge \{ s = \{v\} \cup s' \; ; [x, 2] * x \mapsto v * (x + 1) \mapsto y * lseg(y, s') \}$
}

$\{ lseg^{1} (x, s) \}$ void listfree(loc x) $\{ emp \}$

```
if (x == 0) { } else {

    let nxt2 = *(x + 1);
```

$\{ x \neq 0 \wedge s = \{v\} \cup s' \; ; [x, 2] * x \mapsto v * (x + 1) \mapsto nxt2 * lseg^{1} (nxt2, s') \}$

```
    ??
```

$\{ emp \}$

```
}
```

predicate lseg (**loc** x, **set** s) {
  | $x = 0 \land \{ s = \varnothing \; ; \; emp \}$
  | $x \neq 0 \land \{ s = \{v\} \cup s' \; ; [x, 2] * x \mapsto v * (x + 1) \mapsto y * lseg(y, s') \}$
}

$\{ lseg^{1}(x, s) \}$ `void listfree(loc x)` $\{ emp \}$

```
if (x == 0) { } else {

    let nxt2 = *(x + 1);

    free(x);
```

$\{ x \neq 0 \land s = \{v\} \cup s' \; ; lseg^{1}(nxt2, s') \}$

```
    ??
```

$\{ emp \}$

```
}
```

predicate lseg (loc x, set s) {
| $x = 0 \wedge$ { $s = \varnothing$; emp }
| $x \neq 0 \wedge$ { $s = \{v\} \cup s'$; $[x, 2] * x \mapsto v * (x + 1) \mapsto y * \text{lseg}(y, s')$ }
}

{ $\text{lseg}^1 (x, s)$ } void listfree(loc x) { emp }

```
if (x == 0) { } else {

    let nxt2 = *(x + 1);

    free(x);

    listfree(nxt2);
```

{ $x \neq 0 \wedge s = \{v\} \cup s'$; emp }

```
   ??
```

{ emp }

```
}
```

predicate lseg (loc x, set s) {
  | $x = 0 \land \{ s = \varnothing \;;\; emp \}$
  | $x \neq 0 \land \{ s = \{v\} \cup s' \;;\; [x, 2] * x \mapsto v * (x + 1) \mapsto y * lseg(y, s') \}$
}

$\{ lseg^{1}(x, s) \}$ `void listfree(loc x)` $\{ emp \}$

```
if (x == 0) { } else {
    let nxt2 = *(x + 1);
    free(x);
    listfree(nxt2);
    skip;
}
```

```
void listfree(loc x) {
  if (x == 0) { } else {
    let nxt2 = *(x + 1);
    free(x);
    listfree(nxt2);
  }
}
```

“Unfolding” Predicate Instances in a Postcondition

predicate lseg (**loc** x, **set** s) {

$| \; x = 0 \; \wedge \; \{ s = \emptyset \quad ; \; emp \}$

$| \; x \neq 0 \wedge \{ s = \{v\} \cup s' \; ; [x, 2] * x \mapsto v * (x + 1) \mapsto y * lseg(y, s') \}$

}

$$\{ s = \{v\} \cup s' \quad ; \; lseg^{\mathbf{1}} (y, s') \}$$

? ?

$$\{ lseg^{\mathbf{0}} (z, s) \}$$

```
predicate lseg (loc x, set s) {
  | x = 0  ∧  { s = ∅          ;  emp }
  | x ≠ 0 ∧  { s = {v} ∪ s'   ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg(y, s') }
}
```

$\{\ s = \{v\} \cup s' \ ;\ lseg^{\mathbf{1}}(y, s')\ \}$

??

$\{\ z = 0 \land s = \emptyset ; emp\ \}$

$\Rightarrow$ UNSAT

predicate lseg (**loc** x, **set** s) {

$\mid\ x = 0 \ \wedge\ \{\ s = \emptyset \quad ;\ \mathsf{emp}\ \}$

$\mid\ x \neq 0 \ \wedge\ \{\ s = \{v\} \cup s' \quad ; [x, 2] * x \mapsto v * (x + 1) \mapsto y * \mathsf{lseg}(y, s')\ \}$

}

$$\{\ s = \{v\} \cup s' \quad ;\ \mathsf{lseg}^{\mathbf{1}}(y, s')\ \}$$

??

$$\{\ \mathsf{lseg}^{\mathbf{0}}(z, s)\ \}$$

predicate lseg (**loc** x, **set** s) {

$| \; x = 0 \; \wedge \; \{ s = \emptyset \; ; \; emp \}$

$| \; x \neq 0 \wedge \{ s = \{v\} \cup s' \; ; [x, 2] * x \mapsto v * (x + 1) \mapsto y * lseg(y, s') \}$

}

$$\{ s = \{v\} \cup s' \; ; \; \boxed{lseg^{\mathbf{1}}(y, s')} \; \}$$

??

$$\{ z \neq 0 \wedge s = \{v'\} \cup s'' \; ; [z, 2] * z \mapsto v' * (z + 1) \mapsto z' * \boxed{lseg^{\mathbf{1}}(z', s';)} \}$$

```
predicate lseg (loc x, set s) {
  | x = 0  ∧  { s = ∅         ;  emp }
  | x ≠ 0 ∧  { s = {v} ∪ s'   ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg(y, s') }
}
```

$\{\ s = \{v\} \cup s' \quad ;\ \boxed{lseg^{1}\,(y, s')}\ \}$

??

$\{\ z \neq 0 \land s = \{v'\} \cup s' \quad ;\ [z, 2] * z \mapsto v' * (z + 1) \mapsto y * \boxed{lseg^{1}\,(y, s')}\ \}$

predicate lseg (**loc** x, **set** s) {

$\mid\ x = 0\ \wedge\ \{\ s = \varnothing\ ;\ emp\ \}$

$\mid\ x \neq 0\ \wedge\ \{\ s = \{v\} \cup s'\ ;\ [x, 2] * x \mapsto v * (x + 1) \mapsto y * lseg(y, s')\ \}$

}

$$\{\ s = \{v\} \cup s'\ ;\ emp\ \}$$

??

$$\{\ z \neq 0 \wedge s = \{v'\} \cup s'\ ;\ \boxed{[z, 2] * z \mapsto v' * (z + 1) \mapsto y}\ \}$$

predicate lseg (**loc** x, **set** s) {

| $x = 0 \wedge \{ s = \varnothing \; ; \; emp \}$

| $x \neq 0 \wedge \{ s = \{v\} \cup s' \; ; [x, 2] * x \mapsto v * (x + 1) \mapsto y * lseg(y, s') \}$

}

```
let z = malloc(2);
```

$\{ z \neq 0 \wedge s = \{v\} \cup s' \; ; \; [z, 2] * z \mapsto - * (z + 1) \mapsto - \}$

```
??
```

$\{ z \neq 0 \wedge s = \{v'\} \cup s' \; ; [z, 2] * z \mapsto v' * (z + 1) \mapsto y \}$

predicate lseg (**loc** x, **set** s) {
  | $x = 0 \land \{ s = \varnothing \;;\; \mathrm{emp} \}$
  | $x \neq 0 \land \{ s = \{v\} \cup s' \;;\; [x, 2] * x \mapsto v * (x + 1) \mapsto y * \mathrm{lseg}(y, s') \}$
}

```
let z = malloc(2);
```

$$\{ z \neq 0 \;;\; [z, 2] * z \mapsto - * (z + 1) \mapsto - \}$$

```
??
```

$$\{ z \neq 0 \;;\; [z, 2] * z \mapsto v * (z + 1) \mapsto y \}$$

```
predicate lseg (loc x, set s) {
  | x = 0  ∧  { s = ∅           ;  emp }
  | x ≠ 0 ∧  { s = {v} ∪ s'   ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg(y, s') }
}
```

```
let z = malloc(2);

z := v;
```

$\{\, z \neq 0 \;\; ; (z + 1) \mapsto - \,\}$

```
??
```

$\{\, z \neq 0 \;\; ; (z + 1) \mapsto y \,\}$

```
predicate lseg (loc x, set s) {
  | x = 0  ∧  { s = ∅         ;  emp }
  | x ≠ 0 ∧  { s = {v} ∪ s'   ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg(y, s') }
}
```

```
let z = malloc(2);

z := v;

(z + 1) := y;
```

$\{\ z \neq 0\ \ ;\ emp\ \}$

??

$\{\ z \neq 0\ \ ;\ emp\ \}$

```
predicate lseg (loc x, set s) {
  | x = 0  ∧  { s = ∅          ;  emp }
  | x ≠ 0 ∧  { s = {v} ∪ s'   ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg(y, s') }
}
```

```
let z = malloc(2);

z := v;

(z + 1) := y;

skip
```

```
predicate lseg (loc x, set s) {
  | x = 0  ∧  { s = ∅          ; emp }
  | x ≠ 0 ∧  { s = {v} ∪ s'   ; [x, 2] ∗ x ↦ v ∗ (x + 1) ↦ y ∗ lseg(y, s') }
}
```

$\{\ s = \{v\} \cup s' \ ;\ \mathrm{lseg}^{\mathbf{1}}(y, s')\ \}$

```
let z = malloc(2);

z := v;

(z + 1) := y;
```

$\{\ \mathrm{lseg}^{\mathbf{0}}(z, s)\ \}$

Tags and Termination

- Tags in *preconditions* ensure recursive calls on smaller ***sub-heaps***
    - Recursive calls “seal” their resulting heaps, erasing tags and preventing *“chained”* recursive calls.
- *Predicate instances* in *postconditions* are ***“unfolded”*** to match a pre.
    - Tags in the post *control* the number of *unfoldings*.
    - Infinite unfolding are impossible by design.

Theorem:

If $\mathbf{P} \rightsquigarrow \mathbf{Q} \mid \mathbf{c}$

then **c** terminates.

Obvious Limitation

$\{P * \mathrm{lseg}^{1}(x, s)\}$ `void foo(loc x, loc y)` $\{\mathrm{lseg}^{\blacksquare}(y, s)\}$

$\{x, y, z\}$; $\{P_1 * \mathrm{lseg}^{1}(x, s) * P_2\}$

`??`

$\{\mathrm{lseg}^{1}(z, s)\}$

$\{P * \mathrm{lseg}^{1}(x, s)\}$ `void foo(loc x, loc y)` $\{\mathrm{lseg}^{\blacksquare}(y, s)\}$

```
foo(x, y);
```

$\{\mathrm{lseg}^{\blacksquare}(y, s) * P_2\}$

```
??
```

$\{\mathrm{lseg}^{1}(z, s)\}$

All Rules

StarPartial

$$\frac{\begin{array}{c} x + \iota \neq y + \iota' \notin \phi \qquad \phi' \triangleq \phi \wedge (x + \iota \neq y + \iota') \\ \Sigma; \Gamma;\ \{\phi'; \langle x, \iota \rangle \mapsto e * \langle y, \iota' \rangle \mapsto e' * P\} \leadsto \{Q\} \,|\, c \end{array}}{\Sigma; \Gamma;\ \{\phi; \langle x, \iota \rangle \mapsto e * \langle y, \iota' \rangle \mapsto e' * P\} \leadsto \{Q\} \,|\, c}$$

Alloc

$$\frac{\begin{array}{c} R = [z, n] * \circledast_{0 \leq i \leq n} (\langle z, i \rangle \mapsto e_i) \qquad z \in \mathsf{EV}(\Gamma, \mathcal{P}, \mathcal{Q}) \\ (\{y\} \cup \{\overline{t_i}\}) \cap \mathsf{Vars}(\Gamma, \mathcal{P}, \mathcal{Q}) = \emptyset \\ R' \triangleq [y, n] * \circledast_{0 \leq i \leq n} (\langle y, i \rangle \mapsto t_i) \\ \Sigma; \Gamma;\ \{\phi; P * R'\} \leadsto \{\psi; Q * R\} \,|\, c \end{array}}{\Sigma; \Gamma;\ \{\phi; P\} \leadsto \{\psi; Q * R\} \,|\, \mathbf{let}\ y = \mathtt{malloc}(n); c}$$

Free

$$\frac{\begin{array}{c} R = [x, n] * \circledast_{0 \leq i \leq n} (\langle x, i \rangle \mapsto e_i) \\ \mathsf{Vars}(\{x\} \cup \{\overline{e_i}\}) \subseteq \Gamma \qquad \Sigma; \Gamma;\ \{\phi; P\} \leadsto \{Q\} \,|\, c \end{array}}{\Sigma; \Gamma;\ \{\phi; P * R\} \leadsto \{Q\} \,|\, \mathtt{free}(n); c}$$

Open

$$\frac{\begin{array}{c} \mathcal{D} \triangleq p(\overline{x_i}) \overline{\langle \xi_j, \{\chi_j, R_j\} \rangle}_{j \in 1 \ldots N} \in \Sigma \\ \ell < \mathsf{MaxUnfold} \qquad \sigma \triangleq [\overline{x_i \mapsto y_i}] \qquad \mathsf{Vars}(\overline{y_i}) \subseteq \Gamma \\ \phi_j \triangleq \phi \wedge [\sigma]\xi_j \wedge [\sigma]\chi_j \qquad P_j \triangleq \lceil [\sigma] R_j \rceil^{\ell+1} * \lceil P \rceil \\ \forall j \in 1 \ldots N, \quad \Sigma; \Gamma;\ \{\phi_j; P_j\} \leadsto \{Q\} \,|\, c_j \\ c \triangleq \mathbf{if}\ ([\sigma]\xi_1)\ \{c_1\}\ \mathbf{else}\ \{\mathbf{if}\ ([\sigma]\xi_2) \ldots \mathbf{else}\ \{c_N\}\} \end{array}}{\Sigma; \Gamma;\ \{\phi; P * p^{\ell}(\overline{y_i})\} \leadsto \{Q\} \,|\, c}$$

Write

$$\frac{\mathsf{Vars}(e) \subseteq \Gamma \qquad \Gamma;\ \{\phi; \langle x, \iota \rangle \mapsto e * P\} \leadsto \{\psi; \langle x, \iota \rangle \mapsto e * Q\} \,|\, c}{\Gamma;\ \{\phi; \langle x, \iota \rangle \mapsto e' * P\} \leadsto \{\psi; \langle x, \iota \rangle \mapsto e * Q\} \,|\, *(x + \iota) = e; c}$$

AbduceCall

$$\frac{\begin{array}{c} \mathcal{F} \triangleq f(\overline{x_i}) : \{\phi_f; P_f * F_f\}\{\psi_f; Q_f\} \in \Sigma \\ F_f \text{ has no predicate instances} \qquad [\sigma]P_f = P \\ F_f \neq \mathsf{emp} \qquad F' \triangleq [\sigma]F_f \qquad \Sigma; \Gamma;\ \{\phi; F\} \leadsto \{\phi; F'\} \,|\, c_1 \\ \Sigma; \Gamma;\ \{\phi; P * F' * R\} \leadsto \{Q\} \,|\, c_2 \end{array}}{\Sigma; \Gamma;\ \{\phi; P * F * R\} \leadsto \{Q\} \,|\, c_1; c_2}$$

UnifyHeaps

$$\frac{\begin{array}{c} [\sigma]R' = R \\ \mathsf{frameable}(R') \qquad \emptyset \neq \mathsf{dom}(\sigma) \subseteq \mathsf{EV}(\Gamma, \mathcal{P}, \mathcal{Q}) \\ \Gamma;\ \{P * R\} \leadsto [\sigma]\{\psi; Q * R'\} \,|\, c \end{array}}{\Gamma;\ \{\phi; P * R\} \leadsto \{\psi; Q * R'\} \,|\, c}$$

Frame

$$\frac{\begin{array}{c} \mathsf{EV}(\Gamma, \mathcal{P}, \mathcal{Q}) \cap \mathsf{Vars}(R) = \emptyset \\ \mathsf{frameable}(R') \qquad \Gamma;\ \{\phi; P\} \leadsto \{\psi; Q\} \,|\, c \end{array}}{\Gamma;\ \{\phi; P * R\} \leadsto \{\psi; Q * R\} \,|\, c}$$

Read

$$\frac{\begin{array}{c} a \in \mathsf{GV}(\Gamma, \mathcal{P}, \mathcal{Q}) \qquad y \notin \mathsf{Vars}(\Gamma, \mathcal{P}, \mathcal{Q}) \\ \Gamma \cup \{y\};\ [y/a]\{\phi; \langle x, \iota \rangle \mapsto a * P\} \leadsto [y/a]\{Q\} \,|\, c \end{array}}{\Sigma; \Gamma;\ \{\phi; \langle x, \iota \rangle \mapsto a * P\} \leadsto \{Q\} \,|\, \mathbf{let}\ y = *(x + \iota); c}$$

Induction

$$\frac{\begin{array}{c} f \triangleq \text{goal's name} \\ \overline{x_i} \triangleq \text{goal's formals} \\ P_f \triangleq p^1(\overline{y_i}) * \lceil P \rceil \qquad Q_f \triangleq \lceil Q \rceil \\ \mathcal{F} \triangleq f(\overline{x_i}) : \{\phi_f; P_f\}\{\psi_f; Q_f\} \\ \Sigma, \mathcal{F}; \Gamma;\ \{\phi; p^0(\overline{y_i}) * P\} \leadsto \{Q\} \,|\, c \end{array}}{\Sigma; \Gamma;\ \{\phi; p^0(\overline{y_i}) * P\} \leadsto \{Q\} \,|\, c}$$

Emp

$$\frac{\mathsf{EV}(\Gamma, \mathcal{P}, \mathcal{Q}) = \emptyset \qquad \phi \Rightarrow \psi}{\Gamma;\ \{\phi; \mathsf{emp}\} \leadsto \{\psi; \mathsf{emp}\} \,|\, \mathtt{skip}}$$

NullNotLVal

$$\frac{\begin{array}{c} x \neq 0 \notin \phi \qquad \phi' \triangleq \phi \wedge x \neq 0 \\ \Sigma; \Gamma;\ \{\phi'; \langle x, \iota \rangle \mapsto e * P\} \leadsto \{Q\} \,|\, c \end{array}}{\Sigma; \Gamma;\ \{\phi; \langle x, \iota \rangle \mapsto e * P\} \leadsto \{Q\} \,|\, c}$$

Close

$$\frac{\begin{array}{c} \mathcal{D} \triangleq p(\overline{x_i}) \overline{\langle \xi_j, \{\chi_j, R_j\} \rangle}_{j \in 1 \ldots N} \in \Sigma \\ \ell < \mathsf{MaxUnfold} \qquad \sigma \triangleq [\overline{x_i \mapsto y_i}] \\ \text{for some } k,\ 1 \leq k \leq N \qquad R' \triangleq \lceil [\sigma] R_k \rceil^{\ell+1} \\ \Sigma; \Gamma;\ \{\mathcal{P}\} \leadsto \{\psi \wedge [\sigma]\xi_k \wedge [\sigma]\chi_k; Q * R'\} \,|\, c \end{array}}{\Sigma; \Gamma;\ \{\mathcal{P}\} \leadsto \{\psi; Q * p^{\ell}(\overline{y_i})\} \,|\, c}$$

Inconsistency

$$\frac{\phi \Rightarrow \bot}{\Gamma;\ \{\phi; P\} \leadsto \{Q\} \,|\, \mathtt{error}}$$

SubstLeft

$$\frac{\begin{array}{c} \phi \Rightarrow x = y \\ \Gamma;\ [y/x]\{\phi; P\} \leadsto [y/x]\{Q\} \,|\, c \end{array}}{\Gamma;\ \{\phi; P\} \leadsto \{Q\} \,|\, c}$$

Call

$$\frac{\begin{array}{c} \mathcal{F} \triangleq f(\overline{x_i}) : \{\phi_f; P_f\}\{\psi_f; Q_f\} \in \Sigma \\ R =^{\ell} [\sigma]P_f \qquad \phi \Rightarrow [\sigma]\phi_f \\ \phi' \triangleq [\sigma]\psi_f \qquad R' \triangleq \lceil [\sigma] Q_f \rceil \qquad \overline{e_i} = [\sigma]\overline{x_i} \\ \mathsf{Vars}(\overline{e_i}) \subseteq \Gamma \qquad \Sigma; \Gamma;\ \{\phi \wedge \phi'; P * R'\} \leadsto \{Q\} \,|\, c \end{array}}{\Sigma; \Gamma;\ \{\phi; P * R\} \leadsto \{Q\} \,|\, f(\overline{e_i}); c}$$

Pick

$$\frac{\begin{array}{c} y \in \mathsf{EV}(\Gamma, \mathcal{P}, \mathcal{Q}) \\ \mathsf{Vars}(e) \in \Gamma \cup \mathsf{GV}(\Gamma, \mathcal{P}, \mathcal{Q}) \\ \Gamma;\ \{\phi; P\} \leadsto [e/y]\{\psi; Q\} \,|\, c \end{array}}{\Gamma;\ \{\phi; P\} \leadsto \{\psi; Q\} \,|\, c}$$

UnifyPure

$$\frac{\begin{array}{c} [\sigma]\psi' = \phi' \\ \emptyset \neq \mathsf{dom}(\sigma) \subseteq \mathsf{EV}(\Gamma, \mathcal{P}, \mathcal{Q}) \\ \Gamma;\ \{\mathcal{P}\} \leadsto [\sigma]\{\mathcal{Q}\} \,|\, c \end{array}}{\Gamma;\ \{\phi \wedge \phi'; P\} \leadsto \{\psi \wedge \psi'; Q\} \,|\, c}$$

SubstRight

$$\frac{\begin{array}{c} x \in \mathsf{EV}(\Gamma, \mathcal{P}, \mathcal{Q}) \\ \Sigma; \Gamma;\ \{\mathcal{P}\} \leadsto [e/x]\{\psi, Q\} \,|\, c \end{array}}{\Sigma; \Gamma;\ \{\mathcal{P}\} \leadsto \{\psi \wedge x = e; Q\} \,|\, c}$$

Synthesis Algorithm

Proof Search Algorithm

- Goal-driven, with *backtracking* (in CPS), trying a fixed set of rules;
- *Branching*: some rules (e.g., Close, Unify) emit many alternatives;
- Inductive predicates in the *precondition* emit *more than one subgoal*;
- Along with the program, emits the *complete proof tree*;
- *Conjecture*: the algorithm terminates (to be established formally).

Optimisations

- Invertible Rules (*cf. Focusing* in Proof Theory)
- Partitioning rules into *phases*
- “Early Failure” rules
- Reducing backtracking with symmetry reduction
  - Detecting potentially independent derivations via a version of Frame Rule

“Early Failure” rules

$$\frac{\phi \land \psi \Rightarrow \bot}{\Sigma; \Gamma;\ \{\phi; P\} \leadsto \{\psi, Q\} \,|\, \texttt{magic}}\ \textsc{PostInconsistent}$$

$$\frac{P \text{ has no pred. instances} \qquad \mathsf{EV}(\Gamma, \mathcal{P}, \mathcal{Q}) = \emptyset \quad \neg(\phi \Rightarrow \psi)}{\Sigma; \Gamma;\ \{\phi; P\} \leadsto \{\psi, Q\} \,|\, \texttt{magic}}\ \textsc{PostInvalid}$$

$$\frac{P, Q \text{ have no pred. instances or blocks} \qquad \text{unmachedHeaplets}(P, Q)}{\Sigma; \Gamma;\ \{\phi, P\} \leadsto \{\psi, Q\} \,|\, \texttt{magic}}\ \textsc{UnreachHeap}$$

Implementation

SuSLik

(**S**ynthesis **u**sing **S**eparation **L**og**ik**)

Group	Description	Code	Code/Spec	Time	T-phase	T-inv	T-fail	T-com	T-all	T-IS
Integers	swap two	12	0.9x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	min of two[2]	10	0.7x	0.1	0.1	0.1	< 0.1	0.1	0.2	
Linked List	length[1,2]	21	1.2x	0.4	0.9	0.5	0.4	0.6	1.4	29x
	max[1]	27	1.7x	0.6	0.8	0.5	0.4	0.4	0.8	20x
	min[1]	27	1.7x	0.5	0.9	0.5	0.4	0.5	1.2	49x
	singleton[2]	11	0.8x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	dispose	11	2.8x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	initialize	13	1.4x	< 0.1	0.1	0.1	< 0.1	0.1	< 0.1	
	copy[3]	35	2.5x	0.2	0.3	0.3	0.1	0.2	-	
	append[3]	19	1.1x	0.2	0.3	0.3	0.2	0.3	0.7	
	delete[3]	44	2.6x	0.7	0.5	0.3	0.2	0.3	0.7	
Sorted list	prepend[1]	11	0.3x	0.2	1.4	83.5	0.1	0.1	-	48x
	insert[1]	58	1.2x	4.8	-	-	-	5.0	-	6x
	insertion sort[1]	28	1.3x	1.1	1.8	1.3	1.2	1.2	74.2	82x
Tree	size	38	2.7x	0.2	0.3	0.2	0.2	0.2	0.3	
	dispose	16	4.0x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	copy	55	3.9x	0.4	49.8	-	0.8	1.4	-	
	flatten w/append	48	4.0x	0.4	0.6	0.5	0.4	0.4	0.6	
	flatten w/acc	35	1.9x	0.6	1.7	0.7	0.5	0.6	-	
BST	insert[1]	58	1.2x	31.9	-	-	-	-	-	11x
	rotate left[1]	15	0.1x	37.7	-	-	-	-	-	0.5x
	rotate right[1]	15	0.1x	17.2	-	-	-	-	-	0.8x

[1] From (Qiu and Solar-Lezama 2017) [2] From (Leino and Milicevic 2012) [3] From (Qiu et al. 2013)

Group	*Description*	*Code*	*Code/Spec*	*Time*	*T-phase*	*T-inv*	*T-fail*	*T-com*	*T-all*	*T-IS*
Integers	swap two	12	0.9x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	min of two[2]	10	0.7x	0.1	0.1	0.1	< 0.1	0.1	0.2	
Linked List	length[1,2]	21	1.2x	0.4	0.9	0.5	0.4	0.6	1.4	29x
	max[1]	27	1.7x	0.6	0.8	0.5	0.4	0.4	0.8	20x
	min[1]	27	1.7x	0.5	0.9	0.5	0.4	0.5	1.2	49x
	singleton[2]	11	0.8x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	dispose	11	2.8x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	initialize	13	1.4x	< 0.1	0.1	0.1	< 0.1	0.1	< 0.1	
	copy[3]	35	2.5x	0.2	0.3	0.3	0.1	0.2	-	
	append[3]	19	1.1x	0.2	0.3	0.3	0.2	0.3	0.7	
	delete[3]	44	2.6x	0.7	0.5	0.3	0.2	0.3	0.7	
Sorted list	prepend[1]	11	0.3x	0.2	1.4	83.5	0.1	0.1	-	48x
	insert[1]	58	1.2x	4.8	-	-	-	5.0	-	6x
	insertion sort[1]	28	1.3x	1.1	1.8	1.3	1.2	1.2	74.2	82x
Tree	size	38	2.7x	0.2	0.3	0.2	0.2	0.2	0.3	
	dispose	16	4.0x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	copy	55	3.9x	0.4	49.8	-	0.8	1.4	-	
	flatten w/append	48	4.0x	0.4	0.6	0.5	0.4	0.4	0.6	
	flatten w/acc	35	1.9x	0.6	1.7	0.7	0.5	0.6	-	
BST	insert[1]	58	1.2x	31.9	-	-	-	-	-	11x
	rotate left[1]	15	0.1x	37.7	-	-	-	-	-	0.5x
	rotate right[1]	15	0.1x	17.2	-	-	-	-	-	0.8x

[1] From (Qiu and Solar-Lezama 2017) [2] From (Leino and Milicevic 2012) [3] From (Qiu et al. 2013)

Group	*Description*	*Code*	*Code/Spec*	*Time*	*T-phase*	*T-inv*	*T-fail*	*T-com*	*T-all*	*T-IS*
Integers	swap two	12	0.9x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	min of two[2]	10	0.7x	0.1	0.1	0.1	< 0.1	0.1	0.2	
Linked List	length[1,2]	21	1.2x	0.4	0.9	0.5	0.4	0.6	1.4	29x
	max[1]	27	1.7x	0.6	0.8	0.5	0.4	0.4	0.8	20x
	min[1]	27	1.7x	0.5	0.9	0.5	0.4	0.5	1.2	49x
	singleton[2]	11	0.8x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	dispose	11	2.8x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	initialize	13	1.4x	< 0.1	0.1	0.1	< 0.1	0.1	< 0.1	
	copy[3]	35	2.5x	0.2	0.3	0.3	0.1	0.2	-	
	append[3]	19	1.1x	0.2	0.3	0.3	0.2	0.3	0.7	
	delete[3]	44	2.6x	0.7	0.5	0.3	0.2	0.3	0.7	
Sorted list	prepend[1]	11	0.3x	0.2	1.4	83.5	0.1	0.1	-	48x
	insert[1]	58	1.2x	4.8	-	-	-	5.0	-	6x
	insertion sort[1]	28	1.3x	1.1	1.8	1.3	1.2	1.2	74.2	82x
Tree	size	38	2.7x	0.2	0.3	0.2	0.2	0.2	0.3	
	dispose	16	4.0x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	copy	55	3.9x	0.4	49.8	-	0.8	1.4	-	
	flatten w/append	48	4.0x	0.4	0.6	0.5	0.4	0.4	0.6	
	flatten w/acc	35	1.9x	0.6	1.7	0.7	0.5	0.6	-	
BST	insert[1]	58	1.2x	31.9	-	-	-	-	-	11x
	rotate left[1]	15	0.1x	37.7	-	-	-	-	-	0.5x
	rotate right[1]	15	0.1x	17.2	-	-	-	-	-	0.8x

[1] From (Qiu and Solar-Lezama 2017) [2] From (Leino and Milicevic 2012) [3] From (Qiu et al. 2013)

Group	*Description*	*Code*	*Code/Spec*	*Time*	*T-phase*	*T-inv*	*T-fail*	*T-com*	*T-all*	*T-IS*
Integers	swap two	12	0.9x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	min of two[2]	10	0.7x	0.1	0.1	0.1	< 0.1	0.1	0.2	
Linked List	length[1,2]	21	1.2x	0.4	0.9	0.5	0.4	0.6	1.4	29x
	max[1]	27	1.7x	0.6	0.8	0.5	0.4	0.4	0.8	20x
	min[1]	27	1.7x	0.5	0.9	0.5	0.4	0.5	1.2	49x
	singleton[2]	11	0.8x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	dispose	11	2.8x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	initialize	13	1.4x	< 0.1	0.1	0.1	< 0.1	0.1	< 0.1	
	copy[3]	35	2.5x	0.2	0.3	0.3	0.1	0.2	-	
	append[3]	19	1.1x	0.2	0.3	0.3	0.2	0.3	0.7	
	delete[3]	44	2.6x	0.7	0.5	0.3	0.2	0.3	0.7	
Sorted list	prepend[1]	11	0.3x	0.2	1.4	83.5	0.1	0.1	-	48x
	insert[1]	58	1.2x	4.8	-	-	-	5.0	-	6x
	insertion sort[1]	28	1.3x	1.1	1.8	1.3	1.2	1.2	74.2	82x
Tree	size	38	2.7x	0.2	0.3	0.2	0.2	0.2	0.3	
	dispose	16	4.0x	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	< 0.1	
	copy	55	3.9x	0.4	49.8	-	0.8	1.4	-	
	flatten w/append	48	4.0x	0.4	0.6	0.5	0.4	0.4	0.6	
	flatten w/acc	35	1.9x	0.6	1.7	0.7	0.5	0.6	-	
BST	insert[1]	58	1.2x	31.9	-	-	-	-	-	11x
	rotate left[1]	15	0.1x	37.7	-	-	-	-	-	0.5x
	rotate right[1]	15	0.1x	17.2	-	-	-	-	-	0.8x

[1] From (Qiu and Solar-Lezama 2017) [2] From (Leino and Milicevic 2012) [3] From (Qiu et al. 2013)

ImpSynt *vs* SuSLik

```
loc srtl_insert(loc x, int k)
requires srtl(x)
ensures srtl(ret) ∧
  len(ret) = old(len(x)) + 1 ∧
  min(ret) = (old(k) < old(min(x))
    ? old(k) : old(min(x))) ∧
  max(ret) = (old(max(x)) < old(k)
    ? old(k) : old(max^(x)))
{
  if (cond(1)) {
    loc ?? := new;
    return ??;
  } else {
    statement(1);
    loc ?? := srtl_insert(??, ??);
    statement(1);
    return ??;
  }
}
```

```
{
  0 ≤ n ∧ 0 ≤ k ∧ k ≤ 7 ;
  ret ↦ k * srtl(x, n, lo, hi)
}
void srtl_insert(loc x, loc ret)
{
  n1 = n + 1 ∧
  lo1 = (k ≤ lo ? k : lo) ∧
  hi1 = (hi ≤ k ? k : hi) ;
  ret ↦ y * srtl(y, n1, lo1, hi1)
}
```

Demo

(Do we have time for it?)

Resources

- *Structuring the Synthesis of Heap-Manipulating Programs*
  Nadia Polikarpova and Ilya Sergey
  https://arxiv.org/pdf/1807.07022

- GitHub repository:
  https://github.com/TyGuS/suslik

- Online Demo:
  http://comcom.csail.mit.edu/comcom/#SuSLik

To Take Away

- **Separation Logic (SL)** is a **Proof System** for heap-manipulating programs.
- **Synthetic Separation Logic (SSL)** expresses **program synthesis**
  as algorithmic **proof search** for SL-style specifications.
- **SuSLik** is a *deductive synthesis tool* implementing fast proof search in SSL.
  - Google: "*suslik separation logic*"

Thanks!